CSE 2500-03: Homework 2
Due September 29, 2017
(before start of lecture)

November 28, 2017

1. (10 points) Imagine that *num_orders* and *num_instock* are particular values (as in a computer program. Write a negation for the following (Use De Morgan's Law on the original statement form $(p \wedge q) \vee ((r \wedge s) \wedge t)$):

$$\begin{array}{c}(num\_orders < 50 \text{ and } num\_instock > 300) \\ \text{or } (50 \leq num\_orders < 75 \text{ and } num\_instock > 500).\end{array}$$

2. (10 points) Is the following a tautology? A contradiction? Neither?

$$(\neg p \vee q) \vee (p \wedge \neg p).$$

3. (10 points) Is $(p \oplus p) \vee r \equiv (p \wedge r) \oplus (p \wedge r)$? Justify your answer (using a truth table).

4. (10 points) Show that $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$. Alternatively, that $p \to (q \to r) \equiv (p \wedge q) \to r$.

5. (10 points) Write the following statements in logical form and show whether they are logically equivalent using a truth table and a short explanation.

   If 2 is a factor of n and 3 is a factor of n, then 6 is a factor of n.

   2 is not a factor of n or 3 is not a factor of n or 6 is a factor of n.

6. (10 points) Assume that the statement "If compound X is boiling, then its temperature must be at least 150°C". Which of the following statements is true? Justify your answers.

   (a) Compound X will boil only if its temperature is at least 150°C.

   (b) If compound X is not boiling, then its temperature is at less than 150°C.

   (c) A necessary condition for compound X to boil is that its temperature is at least 150°C.

   (d) A sufficient condition for compound X to boil is that its temperature is at least 150°C.

7. (20 points) You have an encounter with an island with knights (who always tell the truth) and knaves (who always lie). Two natives C and D approach and C says "Both of us are knaves." What is C? What is D?

8. (20 points) You can assume the following statements:

   (a) $p \to q$

   (b) $r \vee s$.

   (c) $\neg s \to \neg t$.

   (d) $\neg q \vee s$

   (e) $\neg s$.

   (f) $(\neg p \wedge r) \to u$.

   (g) $w \vee t$.

   Construct an argument for $u \wedge w$. Do not use a truth table, instead justify each conclusion you make using one of the argument forms introduced in class (in Section 2.3 of book).

1 Suggested Problems

Problems in this section are not required for the homework. They are additional problems that will help you in your progress. If you put forth a good faith effort on all of these problems then you will receive 10 additional points on your homework grade (not to exceed 100).

1. Is the following a tautology? A contradiction? Neither?

$$(p \wedge p) \vee (\neg p \vee (p \wedge \neg q)).$$

2. Show that $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$.

3. Assume x represents a fixed real number. You may assume that $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$. Rewrite the following statement (using the above rule):

   If $x > 2$ or $x < -2$, then $x^2 > 4$.

4. Show that proof by division into cases is a valid argument form. That is, show

$$((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r.$$

5. Give an example of a statement that is not logically equivalent to its converse.

6. Give an example of a statement that is not logically equivalent to its inverse.

7. Convert the following statement to if-then form: "passing the final exam is a necessary condition for passing the course."