

CSE 2500-03: Homework 3
Due October 11, 2017
(before start of lecture)

December 8, 2017

1. (5 points) Rewrite the following statement in the form “ \forall _____ if _____ then _____.”
Any valid argument with true premises has a true conclusion.
2. (5 points) Consider the statement “All integers are rational numbers but some rational numbers are not integers.” Write this statement in the form “ $\forall x$ if _____ then _____, but \exists _____ such that _____.”
3. (10 points) Write negations of the following statements:
 - (a) \forall computers c , c has a CPU.
 - (b) \exists a band b such that b has won at least 10 Grammy awards.
4. (10 points) Consider the statement “There are no simple solutions to life’s problems.” Write an informal negation for the statement, and then write the statement formally using quantifiers and variables. (You do not need to write the negation formally.)
5. (5 points) Write the contrapositive of the statement “If A and B do not have any elements in common, then they are disjoint.” You may assume that A and B are fixed sets.
6. (5 points) True or false? All occurrences of the letter h in this assignment are lower case. Justify your answer.
7. (5 points) Rewrite the negation of the following statement using existential and universal quantifiers (first convert the statement to quantifiers and then negate it). “Having a large income is not a necessary condition for a person to be happy.”
8. (5 points) Let S be the set of students at UConn and let M be the set of all movies. Define the proposition $V(s, m)$ as “student s has seen movie m ”. Rewrite the following statement without using the universal or existential quantifiers. “ $\exists s \in S$ and $\exists t \in S$ such that $s \neq t$ and $\forall m \in M, V(s, m) \rightarrow V(t, m)$.”

9. (10 points) For the following equation determine if the following statements are true:
- (a) For all real numbers x , there exists a real number y such that the equation is true.
 - (b) There exists a real number x , such that for all $y \in \mathbb{R}$, the equation is true.

The equation is " $x^2 - 2xy + y^2 = 0$ ". Justify your answer.

10. (5 points) Use the laws for negating universal and existential statements to justify the following logical equivalences:

- (a) $\neg(\forall x \in D(\forall y \in E(P(x, y)))) \equiv \exists x \in D(\exists y \in E(\neg P(x, y)))$.
- (b) $\neg(\exists x \in D(\exists y \in E(P(x, y)))) \equiv \forall x \in D(\forall y \in E(\neg P(x, y)))$.

11. (5 points) In informal speech most sentences of the form "There is a _____ every _____" are intended to be understood as meaning " \forall _____ \exists _____" even though the existential quantifier *there is* comes before the universal quantifier *every*. This interpretation applies to the following well-known sentences. Rewrite them using quantifiers and variables.

- (a) There is a sucker born every minute.
- (b) There is a time for every purpose under heaven.

12. (5 points) 3.3.41 (f) Is the following statement true or false? Justify your answer. " $\forall x \in \mathbb{Z}^+$ and $\forall y \in \mathbb{Z}^+$, $\exists z \in \mathbb{Z}^+$ such that $z = x - y$."

13. (10 points) Rewrite the statement "No good cars are cheap" in the form $\forall x$, if $P(x)$ then $\neg Q(x)$. Are the following arguments valid (justify):

- (a) No good car is cheap.
A VX Roadster is cheap.
 \therefore A VX Roadster is not good.
- (b) No good car is cheap.
An Omnex is a not a good car.
 \therefore an Omnex is cheap.

14. (5 points) Is the following argument valid (justify)?

Nothing intelligible ever puzzles me.

Logic puzzles me.

\therefore Logic is unintelligible.

15. (10 points) Below we list a set of premises. A single conclusion follows when all the given premises are taken into consideration, it is difficult to see because the premises are jumbled. Reorder the premises to make it clear that a conclusion follows logically and state the valid conclusion.

- (a) All writers who understand human nature are clever.
- (b) No one is a true poet unless he can stir the human heart.
- (c) Shakespeare wrote *Hamlet*.
- (d) No writer who does not understand human nature can stir the human heart.
- (e) None but a true poet could have written *Hamlet*.

1 Suggested Problems

1. Rewrite this statement in if-then form: "Passing a comprehensive exam is a necessary condition for obtaining a master's degree."
2. Rewrite the following two statements in if-then form and say how they are related:
 - (a) All the children in Tom's family are female.
 - (b) All the females in Tom's family are children.
3. Is the following argument valid? Draw pictures of each relevant set to justify:
 - (a) No vegetarians eat meat.
 - (b) All vegans are vegetarians.
 - (c) No vegans eat meat.
4. Is the following argument valid? Draw pictures of each relevant set to justify:
 - (a) All teachers occasionally make mistakes.
 - (b) No god makes a mistake.
 - (c) No teachers are gods.
5. Is the following argument valid? Draw pictures of each relevant set to justify:
 - (a) No college cafeteria food is good.
 - (b) No good food is wasted.
 - (c) No college cafeteria food is wasted.
6. Is the following argument form valid?
 $\forall x$ if $P(x)$ then $Q(x)$.
 $\neg P(a)$ for some a .
Therefore $\neg Q(a)$.
7. Consider the sequence of digits 0204. A person claims that all the 1s in the sequence are to the left of all the 0s in the sequence. Is this true? Justify your answer.