CSE 2500-03: Homework 4
Part 1 Due October 18, 2017
(before start of lecture)

December 26, 2017

1. (10 points) Prove that the following statement is false: "There exists an integer $k \geq 4$ such that $2k^2 - 5k + 2$ is prime." Recall our definition of prime numbers:

   Definition 1. *A number $p \in \mathbb{Z}^+$ is prime if and only if $p > 1$ and $\forall x, y$ such that $xy = p$ either $x = p$ and $y = 1$ or $x = 1$ and $y = p$.*

2. (5 points) Determine if the statement is true or false, if true prove, if false construct a counterexample:

   "If m and n are positive integers and mn is a perfect square, then m and n are both perfect squares."

   Definition 2. *An integer n is called a perfect square if and only if $n = k^2$ for some integer k.*

3. (5 points) Prove the following statement: "Given two rational number r, s where $r < s$ there exists another rational number between r and s."

4. (10 points) Suppose that a, b, c and d are integers and $a \neq c$. Suppose that $x \in \mathbb{R}$ and that
$$\frac{ax + b}{cx + d} = 1.$$
   Is x necessarily rational? If so, prove this fact.

5. (5 points) Use the unique factorization theorem to write the following integers in standard factored form:

   (a) 1176.

   (b) 5733.

   (c) 3675.

6. (10 points) Prove the following statement: "The square of any integer has the form $4k$ or $4k + 1$ for some integer k."