CSE 2500-03: Homework 4
Part 2 Due October 25, 2017
(before start of lecture)

December 26, 2017

7. (10 points) Prove that if m, d and k are integers and $d > 0$, then $(m + dk) \mod d = m \mod d$.

8. (10 points) Prove the following statement. You can use the results from the previous problem.

   For all integers m and n, and a positive integer d, $(m + n) \mod d = ((m \mod d) + (n \mod d)) \mod d$.

   Hint: Suppose b = m mod d, and c = n mod d. Think about how you can relate b and c to m + n.

9. (10 points) Prove the statement by contradiction: "for all integers a if $a \mod 6 = 3$, then $a \mod 3 \neq 2$."

10. (15 points) Prove the statement in two ways by contradiction and by contrapositive: "For all integers m and n, if mn is even then m is even or n is even."

    Hint: Use the definition of even and odd integers and the parity property.

11. (10 points) Prove that "If n is any positive integer that is not a perfect square, then $\sqrt{n}$ is irrational." You may use the fact that even integer has a unique factorization.

    Hint: If an integer n is not a perfect square, some exponent in n's standard factored form is odd (i.e., some prime factor occurs an odd number of times).

1 Suggested Problems

1. Prove that $\sqrt{3}$ is not a rational number.

2. Prove that the sum of two odd integers is even.

3. Below is an incorrect proof that the sum of two rational numbers is rational. Find the mistake in the proof:

Proof: Suppose r and s are rational numbers. By the definition of rational, $r = a/b$ for some integers a and b with $b \neq 0$, and $s = a/b$ for some integers a and b with $b \neq 0$. Then

$$r + s = \frac{a}{b} + \frac{a}{b} = \frac{2a}{b}.$$

Let $p = 2a$ Then p is an integer because it is a product of integers. Hence, $r + s = p/b$, where p and b are integers and $b \neq 0$. Thus, $r + s$ is a rational number. This completes the proof.

4. Suppose that x, y, n are positive integers and $xy \mod n = 0$. Disprove that either $x \mod n = 0$ or $y \mod n = 0$.

5. Prove for all integers n such that $n \mod 5 = 3$ then $n^2 \mod 5 = 4$.