CSE 2500-03: Homework 5
Due November 17, 2017
(before start of lecture)

November 13, 2017

1. (10 points) Show that
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$
for all integers $n \geq 1$ by mathematical induction.

2. (15 points) Prove that
$$\sum_{i=1}^{n+1} i2^i = n\cdot 2^{n+2}+2,$$
for all integers $n \geq 0$ by mathematical induction.

3. (15 points) By mathematical induction, show that $1+nx \leq (1+x)^n$ for all real numbers $x > -1$ and all integers $n \geq 2$.

4. (15 points) A sequence $c_0, c_1, c_2, ...$ is defined by letting $c_0 = 3$ and $c_k = (c_{k-1})^2$ for all integers $k \geq 1$. Show that $c_n = 3^{2^n}$ for all integers $n \geq 0$.

5. (15 points) Suppose that $g_1, g_2, g_3, ...$ is a sequence defined as follows:
$$g_1 = 3, g_2 = 5$$
$$g_k = 3g_{k-1} - 2g_{k-2}, \text{for all } k \geq 3$$
Prove that $g_n = 2^n + 1$ for all integers $n \geq 1$.

6. (15 points) Let $d_0, d_1, d_2, ...$ be defined by the formula $d_n = 3^n - 2^n$ for all integers $n \geq 0$. Show that this sequence satisfies the recurrence relation (whenever $k \geq 2$):
$$d_k = 5d_{k-1} - 6d_{k-2}.$$

7. (15 points) Use iteration to guess an explicit formula for the recurrence relation
$$b_k = \frac{b_{k-1}}{1+b_{k-1}}, \text{for all integers } k \geq 1$$
$$b_0 = 1.$$
Use mathematical induction to show that your guessed formula is correct for all $k \geq 0$.

1 Suggested Problems

1. Consider the sum of the first k odd numbers. Find an explicit formula for the sum and prove this formula using mathematical induction.

2. Show that any set with n elements has 2^n subsets (by mathematical induction).

3. We showed De Morgan's laws for a negation over a conjunction of size 2. Use mathematical induction to extend this to larger size conjunctions. That is, show that

$$\neg(A_1 \wedge A_2 \wedge \cdots \wedge A_n) \equiv (\neg A_1 \vee \neg A_2 \vee \cdots \vee \neg A_n).$$

   where A_i are arbitrary logical statements.

4. Prove by mathematical induction that $5|11^n - 6$ for all positive integers n.

5. Prove by induction that $2^n \geq 2n$ for all integers $n \geq 2$.

6. Show by strong induction that every integer $n \geq 12$ is expressible as a sum $4k + 5\ell$ for integers $k, \ell \geq 0$.