CSE 2500-03: Homework 6
Due December 6, 2017
(before start of lecture)

December 15, 2017

1. (10 points) Let $A = \{n \in \mathbb{Z} | n = 5r \text{ for some integer } r\}$ and $B = \{m \in \mathbb{Z} | m = 20s \text{ for some integer } s\}$. Is $A \subseteq B$?, explain. Is $B \subseteq A$?, explain.

2. (10 points) Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{b, c, e\}$.

   (a) Find $A \cap (B \cup C), (A \cap B) \cup C$ and $(A \cap B) \cup (A \cap C)$. Which of these sets are equal?

   (b) Find $(A - B) - C$ and $A - (B - C)$. Are these two sets equal?

3. (10 points) Let $S_i = \{x \in \mathbb{R} | 1 < x < 1 + \frac{1}{i}\} = (1, 1 + \frac{1}{i})$ for all positive integers i.

   (a) What is $\cup_{i=1}^{\infty} S_i$?

   (b) What is $\cap_{i=1}^{\infty} S_i$?

4. (6 points) Is $\{\{5, 4\}, \{7, 2\}, \{1, 3, 4\}, \{6, 8\}\}$ a partition of $\{1, 2, 3, 4, 5, 6, 7, 8, 8\}$?

   Is $\{\{1, 5\}, \{4, 7\}, \{2, 8, 6, 3\}\}$ a partition of $\{1, 2, 3, 4, 5, 6, 7, 8\}$?

5. (4 points) Find $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$.

6. (10 points) Let $A_1 = \{1, 2, 3\}, A_2 = \{u, v\}$ and $A_3 = \{m, n\}$. Find the following sets:

   (a) $(A_1 \times A_2) \times A_3$.

   (b) $A_1 \times A_2 \times A_3$.

   How are these sets different?

7. (10 points) Use an elementary argument to prove the following statement. You may assume that all subsets are sets of a universal set U. It may be helpful to proceed by division into cases.

   For all sets A, B, and C, if $A \subseteq C$ and $B \subseteq C$ then $A \cup B \subseteq C$.

8. (10 points) Use the element method for proving a set equals the empty set to prove that :

   For all sets A, B, and C, if $A \subseteq B$ and $B \cap C = \emptyset$ then $A \cap C = \emptyset$.

You may assume that all sets are subsets of a universal set U and you may wish to proceed by contradiction.

9. (10 points) Use the element method for proving a set equals the empty set to prove that :

   For all sets A, B, and C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.

   You may assume that all sets are subsets of a universal set U and you may wish to proceed by contradiction.

10. (10 points) Construct an algebraic proof for the following (cite a set identity to justify each step):

    For all sets A, B, and C,

    $$(A - B) - (B - C) = A - B.$$

    Recall that you must show the two sets are equal.

11. (10 points) Simplify the expression citing a set identity in each step:

    $$((A \cap (B \cap C)) \cap (A - B)) \cap (B \cup C^c).$$

1 Suggested Problems

1. Derive the set identity $A \cup (A \cap B) = A$ from other set identities. Start by showing that for all subsets B of a universal set U, $U \cup B = U$.
2. Define $A = \{1, 2, 3\}$. What is $\mathcal{P}(A)$? What is $|\mathcal{P}(A)|$?
3. Show that $|\mathbb{Z}^+| = |\mathbb{Z}|$.
4. Show that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
5. Show that for any A, $A - A = \emptyset$.
6. Define two partitions of the integers where each part of the partition has an infinite number of elements.